

Small-Signal Modulation Bandwidth of Purcell-Enhanced Nanocavity Light Emitters

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Abstract: We present a new analysis of the modulation bandwidth of nanocavity light emitters. The modulation bandwidth is enhanced by the Purcell effect, but only if the device is operated below threshold. The maximum Purcell-enhanced 3-dB bandwidth scales inversely with the modal volume.

Introduction

Semiconductor nanocavities are of interest for their potential as threshold-less lasers and high-speed modulated sources. It has been postulated that the Purcell effect can enhance the modulation speed of nanocavity light emitters (NCLEs) [1,2]. Here, we derive a new analytical expression for the maximum obtainable 3-dB bandwidth of a NCLE, and show that the bandwidth scales inversely with the modal volume. To maximize the bandwidth, the Purcell factor is optimized and this, in turn, determines the optimum cavity Q . For typical device parameters, the optimum Purcell factor is ~ 900 . We conclude that previous papers may have overestimated the modulation bandwidth of NCLEs.

Theory

To accurately represent a NCLE, the classic laser rate equations are modified to include the Purcell factor, F [3]:

$$\frac{dN}{dt} = J - GS - (F-1)R'_{sp} - \frac{N}{\tau_N} \quad \text{and} \quad \frac{dS}{dt} = \left[\Gamma G - \frac{1}{\tau_p} \right] S + \Gamma FR'_{sp} \quad (1,2)$$

where N is the carrier density, S is the photon density, J is the carrier injection rate, G is the optical gain, τ_N is the carrier lifetime, Γ is the confinement factor, and τ_p is the photon lifetime. As shown in (1,2), the spontaneous emission rate into the lasing mode, R'_{sp} , is enhanced by the Purcell factor, F [4,5]. The Purcell factor is given by $F = 2p\Gamma_r Q / \pi^2 V_n$, where p is the polarization anisotropy factor, Γ_r is the relative confinement factor, and Q is the cavity quality factor. The normalized modal volume is $V_n = V_p / (\lambda/2n)^3$, where V_p is the modal volume, λ is the lasing wavelength, and n is the effective index of the cavity mode. As shown below, the normalized modal volume is a key parameter that strongly influences the modulation bandwidth.

Using linear models of the gain and spontaneous emission, and solving the steady-state solution for (2), we obtain a relationship between the steady-state carrier and photon densities, N_0 and S_0 respectively:

$$N_0 \approx N_{th0} S_0 V_p / (S_0 V_p + \kappa F), \quad (3)$$

where $S_0 V_p$ is the cavity photon number, $\kappa \approx 1/4$ is a fitting parameter used in our linearized gain model, and the classical threshold carrier density N_{th0} is given by $N_{th0} = N_{tr} (1 + 1/\Gamma g_1 N_{tr} \tau_p)$, where g_1 is the differential gain. A possible definition of lasing threshold is when the stimulated emission just exceeds the spontaneous emission: $\Gamma G S_0 = \Gamma F R'_{sp}$. Using a linear gain model, the threshold condition is

$$N_{th} = (N_{tr} + N_{th0})/2 \quad \text{and} \quad S_{th} V_p = \kappa F (N_{th0} + N_{tr}) / (N_{th0} - N_{tr}). \quad (4,5)$$

Equation (4) differs from the classical threshold by less than a factor of 2. Above threshold, N_0 in (3) approaches the classical threshold density N_{th0} . Well below threshold, the carrier density is proportional to the photon density: $N_0 \approx N_{th0} S_0 V_p / \kappa F$. For low- Q NCLEs, N_0 is less than N_{th0} , even at high bias currents. This means that it may be impossible to achieve the threshold photon density. However, we show here that NCLEs need not be lasing to achieve ultra-high bandwidths.

When $\kappa F \gg S_0 V_p$, the Purcell-enhanced spontaneous emission dominates over the stimulated emission. By (5), this can only occur below threshold. In this regime, the resonance frequency ω_R is dominated by the spontaneous emission dynamics, rather than the classical stimulated emission dynamics, yielding

$$\omega_R^2 \approx (aS_0 + rF) / \tau_p \approx rF / \tau_p \quad (6)$$

where $a \equiv \partial G/\partial N$ and $r \equiv \partial R'_{sp}/\partial N$. The damping factor γ is also dominated by the spontaneous emission dynamics:

$$\gamma \approx rF \left[1 + \frac{\varepsilon S_0 V_p}{g_1 \tau_p F \kappa} \right] + \frac{1}{\tau_p} \approx rF \left[1 + \frac{V_p}{g_1 \tau_p F \kappa} (1 + 2\varepsilon S_0) \right], \quad (7)$$

where $r \approx g_1 \kappa V_p (1 + \varepsilon S_0)$. It can be shown that the modulation response of the NCLE is always damping-limited in this regime. Therefore, the 3-dB bandwidth can be approximated as $f_{3dB} \approx \omega_R^2/2\pi\gamma$ and the 3-dB bandwidth is

$$f_{3dB,max} \approx \left\{ 2\pi\tau_p \left[1 + \frac{V_p}{g_1 \tau_p F \kappa} (1 + 2\varepsilon S_0) \right] \right\}^{-1} = \frac{f_0}{Q} \left[1 + \delta^2 \left(\frac{V_n}{Q} \right)^2 (1 + 2\varepsilon S_0) \right]^{-1}. \quad (8)$$

where $\delta^2 \equiv (\pi \lambda/2n)^3 f_0/g_1 \kappa p \Gamma_r$ and $f_0 = \omega_0/2\pi$ is the optical carrier frequency. Fig. 1(a) shows a contour plot of the analytical solution for $f_{3dB,max}$ on the Q - V_n plane (neglecting gain compression). By differentiating (8) with respect to Q , we can find Q_{opt} , the optimal Q at the maximum bandwidth for a fixed V_n : $Q_{opt} = \tau_{p,opt} \omega_0 = \delta V_n$. This Q_{opt} is the dashed line in Fig. 1(a). Therefore, the optimal bandwidth is achieved for a constant Q/V_n ratio. Hence, there is an optimal Purcell factor. Using typical numbers, the optimal Purcell factor is $F_{opt} \approx 2p\Gamma_r \delta/\pi^2 \approx 900$.

From the above, we can express the maximum 3-dB bandwidth as a function of modal volume:

$$f_{3dB,opt} \approx f_0/2\delta V_n. \quad (9)$$

To achieve maximum bandwidth, the parameter δ and the modal volume must be minimized. Therefore, large differential gain, low transparency carrier density, and small modal volume are desired. In Fig. 1(b), we compare the analytical optimal f_{3dB} in (9) as a function of V_n to the numerical value calculated from the full rate equations (1,2) [6]. The parameters used are: $\Gamma = 0.16$, $\lambda = 1.55 \mu\text{m}$, $\varepsilon = 1.5 \times 10^{-23} \text{ m}^3$, $n = 3.5$, $\tau_N = 5.2 \text{ ns}$, $N_{tr} = 1.2 \times 10^{24} \text{ m}^{-3}$, $g_1 = 1.4 \times 10^{-11} \text{ m}^3 \text{ s}^{-1}$, $\kappa = 0.25$, $p = 1$, and $\Gamma_r = 1$. Note that for $V_n > 0.5$, the bandwidth is limited by classical terms and the curves diverge. But for $V_n < 0.5$, where the bandwidth is Purcell-enhanced, the agreement is good.

As pointed out above, to maximize the bandwidth, it is necessary to control the cavity Q . Fig. 1(c) compares the analytical optimal value of Q (Q_{opt}) with the numerical value of Q_{opt} obtained from simulations [6].

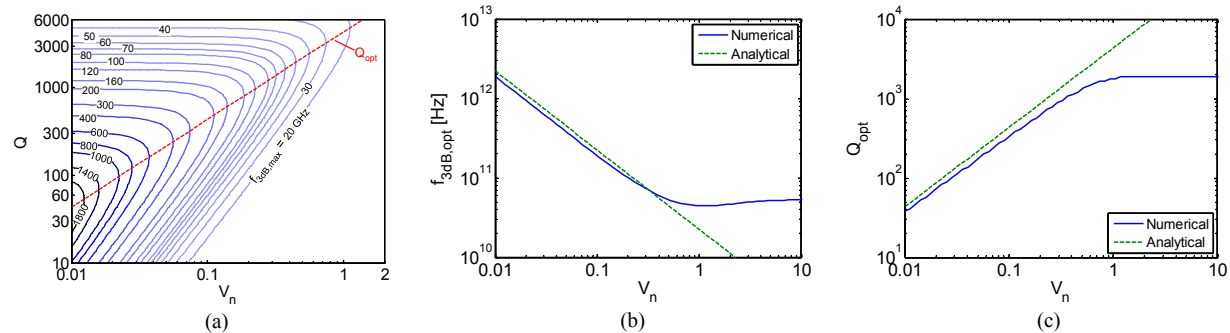


Fig. 1. (a) Contour plot of $f_{3dB,max}$ (in GHz) on the Q - V_n plane. Dotted line gives Q_{opt} for the optimum 3-dB bandwidth as a function of V_n . (b) Numerical and analytical $f_{3dB,opt}$ as a function of V_n . (c) Numerical and analytical Q_{opt} as a function of V_n .

Conclusions

The key results of this paper are the analytical expressions (8) and (9) for the maximum 3-dB bandwidth of Purcell-enhanced nanocavity light emitters. The Purcell effect enhances the bandwidth when the spontaneous emission exceeds the stimulated emission. Therefore, bandwidth enhancement occurs only in the sub-threshold regime, where there is no lasing. Maximum bandwidth is achieved by minimizing the modal volume and by optimizing the cavity Q . The optimum Purcell factor is ~ 900 . We predict maximum bandwidths of ~ 50 GHz and ~ 200 GHz at normalized modal volumes of 0.5 and 0.1, respectively. This is somewhat smaller than previously estimated [2]. The maximum bandwidth is limited by the material gain, the transparency carrier density and the modal volume. While NCLEs may have high bandwidth, they may not benefit from reduced noise, linewidth, and other properties usually found in lasers. Nevertheless, NCLEs may be attractive for applications where coherence requirements are relaxed.

References

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